A Toroidal Model of Electromagnetic Wave Propagation

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Abstract

Electromagnetic waves are usually represented by drawings in which electric and magnetic fields oscillate while the wave travels through space. Such drawings describe the variation of the fields, but they do not necessarily show the complete three-dimensional shape of an electromagnetic wave.

This paper explores a simple question: what could the electric and magnetic fields of a localized electromagnetic wave look like in three-dimensional space?

A toroidal, or ring-shaped, arrangement is considered as one possible geometry. In such a structure, magnetic field lines can form closed paths, while the electric and magnetic fields together define the direction in which electromagnetic energy propagates.

The paper then considers how such a structure could change while the wave travels through space. Particular attention is given to Maxwell's displacement current, which appears whenever the electric field changes with time. This leads to a model in which the electromagnetic field does not simply move through space as an unchanged object, but continuously changes its geometrical form.

The discussion also examines how energy could be distributed within such a field structure, how neighboring structures could interact in coherent light, and whether these ideas might help provide a geometrical picture of some properties usually associated with the quantum behavior of light.

The aim of the paper is not to replace Maxwell's equations, but to investigate a possible three-dimensional physical interpretation of the electromagnetic processes described by them.

1. Introduction

Electromagnetic waves are commonly described by Maxwell's equations in terms of electric and magnetic fields that vary in space and time. These equations provide a precise local description of the mutual evolution of the fields,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

together with the vacuum conditions

$$\nabla \cdot \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0$$

Although these equations determine the local dynamics of electromagnetic fields, they do not by themselves prescribe a unique three-dimensional geometry for a spatially localized electromagnetic structure. Plane waves, spherical waves, beams, and wave packets are different global configurations compatible with the same local field equations.

This raises a simple geometrical question: can a localized electromagnetic wave be represented as a self-organized three-dimensional field structure? The condition

$$\nabla \cdot \mathbf{B} = 0$$

requires the magnetic field to be source-free. For a localized structure, this makes closed magnetic-field configurations a natural possibility. A toroidal geometry is therefore considered in this paper as one candidate organization of the electromagnetic field. It is not implied uniquely by Maxwell's equations, but it provides a simple geometry in which closed magnetic-field paths, spatial localization, and a preferred propagation axis can coexist.

The proposed toroidal picture is not intended as a replacement for Maxwell's electrodynamics. Maxwell's equations remain the local dynamical framework. The additional hypothesis concerns the global spatial organization of the fields and the possibility that electromagnetic stresses may allow such a structure to remain dynamically bounded.

A second question arises from the time dependence of the electric field. Maxwell's displacement-current density is

$$\mathbf{J}_D = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

The displacement current is therefore intrinsically connected with the changing electric field and with the magnetic field produced through the Maxwell–Ampère relation. This suggests that it may play an important role in the internal evolution of a localized electromagnetic structure during propagation.

The purpose of this paper is to examine these possibilities step by step. First, a toroidal organization of the electromagnetic field is considered. The possible dynamical equilibrium of such a structure is then discussed. The role of displacement current and its associated magnetic field is examined as a possible complementary field configuration. Finally, the consequences for energy conservation, propagation, coherence, and discrete energy transfer are explored.

Throughout the analysis, a distinction is maintained between relations that follow directly from Maxwell's equations and additional assumptions introduced by the proposed geometrical model. The central objective is therefore not to assume a toroidal structure as an established description of light, but to determine how far such a representation can be developed consistently within electromagnetic field theory.

2. Toroidal Geometry of the Electromagnetic Field

Maxwell's equations determine the local spatial and temporal relations between electric and magnetic fields, but they do not assign a unique global geometry to a localized electromagnetic structure. The present model therefore asks whether a toroidal configuration can provide a possible three-dimensional organization of the fields while remaining compatible with Maxwellian electrodynamics.

A fundamental property of the magnetic field is

$$\nabla \cdot \mathbf{B} = 0.$$

This equation expresses the absence of magnetic sources and sinks. Magnetic field lines therefore do not begin or end at isolated points in free space. For a spatially localized electromagnetic structure, this makes a closed magnetic-field topology a natural possibility.

A toroidal configuration is one of the simplest geometries in which such closure can occur continuously. It also possesses a well-defined symmetry axis, which can be associated with the direction of propagation. For these reasons, the torus is adopted here as a candidate global geometry of a localized electromagnetic field.

It is important to emphasize that

$$\nabla \cdot \mathbf{B} = 0.$$

does not uniquely imply a toroidal geometry. Many other field configurations satisfy the same condition. The torus is therefore introduced as an geometrical hypothesis whose physical relevance must be tested by the remaining Maxwell equations, energy conservation, stability, and ultimately experiment.

The electric and magnetic fields of such a structure must satisfy

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

and

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

The proposed torus should consequently not be imagined as a static magnetic ring accompanied by an independent electric field. It represents a dynamic electromagnetic configuration in which the spatial variation of each field is continuously related to the temporal variation of the other.

Figure 1 illustrates the proposed geometrical arrangement.

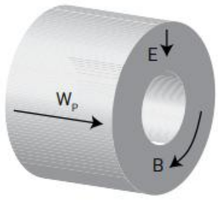

Figure 1*. Proposed toroidal organization of the electromagnetic field. The electric and magnetic fields form a three-dimensional field structure, while the Poynting vector defines the direction of electromagnetic energy transport along the symmetry axis.*

For the proposed orientation of the fields, the direction of electromagnetic energy transport is described by the Poynting vector,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}.$$

The local orientations of E and H are chosen such that S has a component directed along the symmetry axis of the torus. The closed transverse organization of the fields is therefore compatible, at least geometrically, with longitudinal energy transport.

This distinction is useful when comparing the toroidal model with the familiar sinusoidal drawing of an electromagnetic wave. Such a drawing normally shows how selected components of E and B vary with position or time. It does not necessarily represent the complete three-dimensional topology of a localized wave. The toroidal model concerns this additional geometrical question.

An ideal geometrical torus can be characterized by a major radius R and a minor radius r. Its geometrical volume is

$$V_T = 2\pi^2 R r^2.$$

An electromagnetic field, however, has no sharp material surface at which its magnitude suddenly becomes zero. In the present model, V_T must therefore be understood as an effective field volume. A rigorous description would ultimately require explicit spatial functions

$$\mathbf{E}(\mathbf{r}, t) \qquad \text{and} \qquad \mathbf{B}(\mathbf{r}, t)$$

throughout the complete field region.

The toroidal hypothesis therefore establishes only a proposed global geometry. It does not yet explain why such a configuration should remain localized or stable. If a self-contained toroidal structure exists, its electromagnetic stresses must prevent both unlimited expansion and collapse. The question of this possible dynamic equilibrium is considered in the following section.

3. Dynamic Equilibrium of the Toroidal Structure

A localized toroidal electromagnetic structure can persist only if its geometry is dynamically stable. The present model considers the possibility that such stability may arise from the electromagnetic stresses of the electric and magnetic fields themselves, without requiring an external confining mechanism.

The magnetic field possesses an energy density

$$u_B = \frac{B^2}{2\mu_0},$$

and electromagnetic fields can transmit mechanical stress, quantitatively described by the Maxwell stress tensor. Curved magnetic-field configurations exhibit a tension-like contribution directed along the field lines. For a closed magnetic geometry, this suggests a possible contractive tendency associated with the curvature of the magnetic field.

The electric field also contributes to the electromagnetic stress. Its energy density is

$$u_E = \frac{\varepsilon_0 E^2}{2}.$$

For the field geometry proposed in this model, we consider the possibility that the electric contribution produces an opposing expansive tendency. A stable toroidal configuration could then arise if the contractive and expansive electromagnetic stresses balance at a particular geometry.

This proposed balance is illustrated schematically in Figure 2.

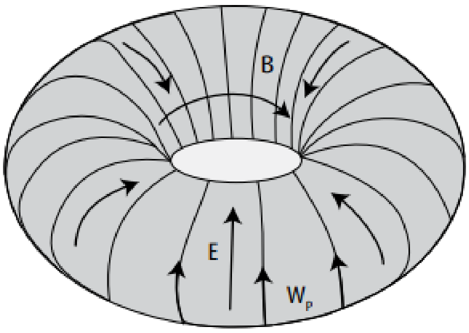

Figure 2. Schematic representation of the proposed dynamic balance within the toroidal structure. The curved magnetic-field configuration is associated with a contractive tendency, while the electric-field configuration is assumed to provide an opposing expansive contribution.

If the effective radial contributions are represented by FB and FE, the proposed equilibrium condition can be written schematically as

$$F_E(R_0) + F_B(R_0) = 0,$$

where R_0 denotes an equilibrium major radius. Taking the outward direction as positive,

$$F_E > 0, \qquad F_B < 0.$$

At equilibrium,

$$F_E = -F_B.$$

These quantities should not yet be interpreted as independently derived forces acting on a material boundary. They represent the net radial effects that would have to emerge from the electromagnetic stress distribution of the complete field configuration.

A more rigorous formulation can be expressed in terms of the total electromagnetic energy. Let

$$W(R) = W_E(R) + W_B(R)$$

denote the energy of a family of toroidal field configurations characterized by the geometrical parameter R. A stationary configuration would satisfy

$$\left.\frac{dW}{dR}\right|_{R=R_0} = 0.$$

For this configuration to represent a stable equilibrium, a small deformation must increase the energy, requiring

$$\left.\frac{d^2W}{dR^2}\right|_{R=R_0} > 0.$$

The energy corresponding to this preferred configuration will be denoted by

$$W_0 = W(R_0).$$

In the present model, W_0 therefore represents a hypothetical preferred structural energy. It is not introduced as a quantum of electromagnetic energy, nor is its existence yet derived from Maxwell's equations. Establishing such an energy minimum requires an explicit three-dimensional field solution and evaluation of its electromagnetic stresses.

This distinction is important because dynamic stability and propagation are not necessarily the same condition. A structure displaced from its preferred configuration may have

$$W \neq W_0$$

and may therefore be non-optimal without necessarily being unable to propagate. Whether propagation itself selects a particular amplitude or energy will be examined separately in later sections.

The proposed equilibrium is also compatible with continuing internal electromagnetic dynamics. The fields need not be static at R_0. They remain subject to Maxwell's relations,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Thus, R_0 should be understood as a stable characteristic geometry of an oscillating electromagnetic configuration rather than as a static arrangement of fields.

The hypothesis of dynamic equilibrium therefore introduces a possible preferred geometry and associated energy W_0, but it does not yet establish either quantitatively. The next question is what happens to the field configuration during the electromagnetic oscillation itself. In particular, the time variation of the electric field necessarily produces Maxwell's displacement current, which provides the starting point for the complementary configuration considered in the following section.

4. Displacement Current as an Intermediate Field Configuration

The preceding sections introduced a possible toroidal geometry and a hypothetical mechanism for its dynamic stability. The next question concerns what happens to this configuration while the electromagnetic fields oscillate.

Maxwell's equations provide a natural starting point. A time-varying electric field is associated with the displacement-current density

$$\mathbf{J}_D = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Unlike conduction current, displacement current does not represent the transport of electric charges through the region.

Nevertheless, it enters the Maxwell–Ampère equation in the same way as conduction current,

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}_D = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Thus, a changing electric field is necessarily associated with magnetic circulation.

Consider a sinusoidally varying electric-field component,

$$E(t) = E_0 \cos \omega t.$$

The corresponding displacement-current density is

$$J_D(t) = -\varepsilon_0 \omega E_0 \sin \omega t.$$

The displacement current is therefore shifted by one quarter of a period relative to the electric field. When the magnitude of the electric field is maximal,

$$|E| = E_0,$$

its time derivative vanishes and

$$J_D = 0.$$

One quarter-period later,

$$E = 0,$$

while the magnitude of the displacement-current density reaches

$$|J_D| = \varepsilon_0 \omega E_0 = J_{D,\max}.$$

The corresponding squared quantities therefore vary as

$$E^2(t) \propto \cos^2 \omega t,$$

$$J_D^2(t) \propto \sin^2 \omega t.$$

This quarter-period relation follows directly from Maxwell's definition of displacement current. Its geometrical interpretation, however, requires an additional assumption.

Proposed cylindrical configuration

The present model considers the possibility that, during the phase in which the displacement current becomes dominant, it is organized approximately along the propagation axis over a finite transverse region. Such a distribution can be represented schematically as an effective cylindrical configuration.

If A_C denotes its effective transverse area, the displacement-current density can be integrated over this area,

$$I_D = \int_{A_C} \mathbf{J}_D \cdot d\mathbf{A}.$$

For an approximately uniform distribution,

$$I_D \simeq J_D A_C.$$

The resulting quantity I_D has the dimensions of current, although no material charges are required to cross the area.

The significance of I_D in the present model lies in the magnetic field associated with the changing electric field. Under the assumed approximate cylindrical symmetry, the Maxwell–Ampère equation gives

$$\oint \mathbf{B}_I \cdot d\mathbf{l} = \mu_0 I_D.$$

For a circular integration path outside the effective current region,

$$B_I(r) = \frac{\mu_0 I_D}{2\pi r}.$$

The combination of the displacement-current distribution and its associated magnetic field will therefore be denoted schematically by

$$(I_D, B_I)$$

The word *cylinder* refers only to the proposed dominant spatial organization of this phase. It does not imply a material cylinder or a sharply bounded electromagnetic object.

Two complementary field configurations

The model thus distinguishes two geometrical descriptions during the oscillatory process:

$$(E, B_T)_{\text{torus}},$$

and

$$(I_D, B_I)_{\text{cylinder}}.$$

This leads to the central geometrical hypothesis

$$\boxed{(E, B_T)_{\text{torus}} \rightleftarrows (I_D, B_I)_{\text{cylinder}}.}$$

The first configuration represents the proposed toroidal organization of the electric and magnetic fields. The second represents the phase in which the displacement-current distribution and its associated magnetic field become dominant.

This distinction should not be interpreted as the existence of two independent magnetic fields. There is only one total electromagnetic field. The notation B_T and B_I is introduced to distinguish contributions associated with the two proposed geometrical phases. A complete three-dimensional solution would ultimately have to describe them as parts of a single continuous function,

$$\mathbf{B}(\mathbf{r}, t).$$

The degree to which the two proposed configurations overlap spatially is not yet known. They may coexist over part of the oscillation, and their magnetic contributions may locally reinforce or oppose one another. Determining this spatial relationship requires an explicit three-dimensional Maxwell solution.

Temporal and spatial progression

The quarter-period phase relation also suggests a characteristic spatial separation during propagation. If the electromagnetic process advances with velocity ccc, then

$$\Delta t = \frac{T}{4}$$

corresponds to

$$\Delta x = c\Delta t = \frac{cT}{4} = \frac{\lambda}{4}.$$

The proposed transformation can therefore be represented schematically as a progression along the propagation direction,

$$T \xrightarrow{\lambda/4} C \xrightarrow{\lambda/4} -T \xrightarrow{\lambda/4} -C \xrightarrow{\lambda/4} T,$$

where T denotes the toroidal configuration, C the displacement-current configuration, and the signs indicate the reversal of field orientation during the oscillation.

This picture differs from that of a rigid toroidal object moving unchanged through space. Instead, the proposed electromagnetic structure is continuously reconstructed as the oscillation advances.

Figure 3 illustrates this interpretation.

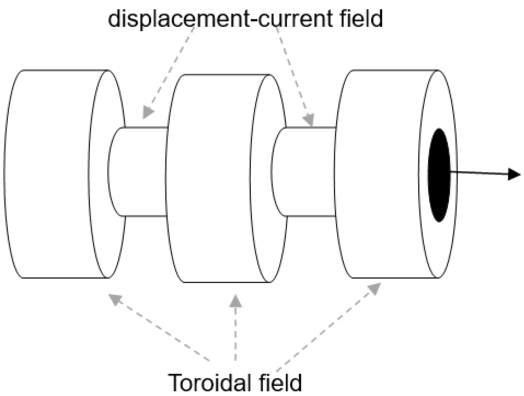

***Figure 3**. Schematic representation of the proposed propagation mechanism. The toroidal field configuration is assumed to transform periodically into an intermediate displacement-current configuration and to be reconstructed farther along the propagation axis. The drawing represents successive phases of one electromagnetic process rather than rigid objects transported through space.*

The Poynting vector,

$$\mathbf{S} = \mathbf{E} \times \mathbf{H},$$

continues to describe electromagnetic energy flux. It should therefore not be identified with the intermediate configuration itself. In the proposed model, the displacement-current distribution and its associated magnetic field describe the intermediate geometrical state, whereas the Poynting vector describes the direction and magnitude of electromagnetic energy transport.

The quarter-period relation between E and JD is a direct consequence of Maxwell's equations. The interpretation of this relation as a periodic transformation between toroidal and cylindrical field geometries is the additional hypothesis introduced here. Whether these configurations can exchange energy while maintaining constant total electromagnetic energy is examined in the following section.

5. Periodic Energy Exchange

The preceding section introduced the hypothesis that electromagnetic propagation may involve a periodic transformation between a toroidal field configuration and an intermediate configuration associated with displacement current. If these are two phases of one continuous electromagnetic process, the next question concerns the distribution of energy between them.

In vacuum, the electromagnetic energy density is

$$u_{\mathrm{EM}} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

For a propagating electromagnetic field in vacuum, the electric and magnetic contributions are related by

$$E = cB,$$

so that

$$\frac{\varepsilon_0 E^2}{2} = \frac{B^2}{2\mu_0}.$$

The total local electromagnetic energy density can therefore be written as

$$u_{\mathrm{EM}} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}.$$

For a sinusoidal field component,

$$E(t) = E_0 \cos \omega t,$$

the corresponding quadratic energy term varies as

$$E^2(t) = E_0^2 \cos^2 \omega t.$$

If the effective geometry of the toroidal phase remains approximately fixed during this part of the cycle, its energy can be represented schematically as

$$W_T(t) = W_{T0} \cos^2 \omega t.$$

Here W_{T0} denotes the maximum energy associated with the toroidal configuration.

At the same time, the displacement-current density derived in the preceding section is

$$J_D(t) = -\varepsilon_0 \omega E_0 \sin \omega t.$$

Hence,

$$J_D^2(t) = \varepsilon_0^2 \omega^2 E_0^2 \sin^2 \omega t.$$

The squared displacement-current term therefore has exactly the complementary temporal dependence,

$$J_D^2(t) \propto \sin^2 \omega t.$$

This observation motivates the central energetic hypothesis of the model: the electromagnetic field associated with the displacement-current configuration may contain the energy that decreases in the toroidal configuration during the transformation cycle.

If the maximum energy of this complementary configuration is denoted by W_{C0}, its temporal dependence may be written in the form

$$W_C(t) = W_{C0} \sin^2 \omega t.$$

The total energy represented by the two configurations is then

$$W(t) = W_{T0} \cos^2 \omega t + W_{C0} \sin^2 \omega t.$$

For this quantity to remain constant throughout the complete oscillation, the two maximum energies must be equal:

$$W_{T0} = W_{C0}.$$

Denoting their common value by W,

$$W_{T0} = W_{C0} = W,$$

gives

$$W(t) = W \left(\cos^2 \omega t + \sin^2 \omega t\right),$$

and therefore

$$\boxed{W(t) = W = \text{const.}}$$

The proposed transformation can consequently be represented schematically as

$$W_T \rightleftarrows W_C,$$

with

$$W_T(t) = W \cos^2 \omega t,$$

and

$$W_C(t) = W \sin^2 \omega t.$$

At

$\omega t = 0,$

the toroidal contribution is maximal,

$$W_T = W, \qquad W_C = 0.$$

One eighth of a period later,

$$\omega t = \frac{\pi}{4},$$

the two contributions are equal,

$$W_T = W_C = \frac{W}{2}.$$

After one quarter of a period,

$$\omega t = \frac{\pi}{2},$$

the complementary configuration is maximal,

$$W_T = 0, \qquad W_C = W.$$

The cycle then reverses continuously.

It is important to clarify the physical meaning of W_C. Displacement current itself is not introduced here as a new fundamental form of energy with an independent energy density proportional simply to J_D^2. In Maxwellian electrodynamics,

$$\mathbf{J}_D = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

describes the temporal variation of the electric field. The energy must still be contained in the complete electric and magnetic field configuration and must ultimately be calculated from

$$u_{\text{EM}} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

Thus, W_C denotes the electromagnetic energy associated with the proposed intermediate field geometry, including the magnetic field B_I generated through the Maxwell–Ampère relation.

This distinction also prevents an incorrect interpretation of the process as

electromagnetic energy →
non-electromagnetic energy →
electromagnetic energy

Both proposed phases remain electromagnetic. What changes is their spatial organization.

The model therefore proposes a periodic redistribution of electromagnetic energy between two geometrical configurations:

$$\boxed{(E, B_T)_{\text{torus}} \rightleftarrows (I_D, B_I)_{\text{cylinder}}.}$$

The temporal complementarity

$$\cos^2 \omega t + \sin^2 \omega t = 1$$

shows that such an energy-conserving exchange is mathematically possible. It does not, however, prove that the proposed cylindrical configuration contains exactly the energy required for

$$W_{T0} = W_{C0}.$$

That equality must be tested by calculating the electromagnetic energy associated with the assumed geometry.

This leads directly to the next question: what geometrical relationship between the toroidal and cylindrical configurations is required for their maximum energies to be equal? The following section examines this condition quantitatively.

6. Geometrical Condition for Energy Conservation

The preceding section established the condition required for complete periodic energy exchange between the two proposed configurations. If the toroidal and displacement-current phases represent complementary stages of the same electromagnetic process, their maximum energies must satisfy

$$\boxed{W_{T0} = W_{C0}.}$$

This section examines what geometrical relation follows from this requirement within a simplified field model.

6.1 Energy of the toroidal configuration

The electromagnetic energy density in vacuum is

$$u_{\text{EM}} = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}.$$

A rigorous calculation of the toroidal energy would require explicit three-dimensional functions

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t),$$

and integration over the complete field region,

$$W_T = \int \left(\frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) dV.$$

Such a solution has not yet been obtained. For the present first-order estimate, we assume that the characteristic electric and magnetic amplitudes satisfy approximately the vacuum-wave relation

$$E_0 \simeq cB_0$$

through the effective toroidal volume V_T.

Under this approximation, the maximum electric and magnetic energy densities are equal,

$$\frac{\varepsilon_0 E_0^2}{2} \simeq \frac{B_0^2}{2\mu_0},$$

and the maximum toroidal energy can be represented as

$$\boxed{W_{T0} \simeq \varepsilon_0 E_0^2 V_T.}$$

Here V_T is an effective field volume rather than the sharply bounded volume of a material object.

6.2 Energy associated with the displacement-current configuration

For

$$E(t) = E_0 \cos \omega t,$$

the maximum displacement-current density is

$$J_{D0} = \varepsilon_0 \omega E_0.$$

Consider an idealized cylindrical region of radius a and length L, with approximately uniform J_{D0} across its transverse area.

Inside this region, the Maxwell–Ampère relation gives

$$B_I(r) = \frac{\mu_0 J_{D0} r}{2}, \qquad 0 \le r \le a.$$

The magnetic energy contained inside the cylindrical region is therefore

$$W_{C0}^{(\mathrm{in})} = \int_0^a \frac{B_I^2(r)}{2\mu_0}\, 2\pi r L\, dr.$$

Substituting BI(r) gives

$$W_{C0}^{(\mathrm{in})} = \frac{\mu_0 J_{D0}^2 \pi L}{4} \int_0^a r^3\, dr,$$

and therefore

$$\boxed{W_{C0}^{(\mathrm{in})} = \frac{\mu_0 J_{D0}^2 \pi L a^4}{16}.}$$

Using

$$J_{D0} = \varepsilon_0 \omega E_0,$$

we obtain

$$W_{C0}^{(\mathrm{in})} = \frac{\mu_0 \varepsilon_0^2 \omega^2 E_0^2 \pi L a^4}{16}.$$

Since

$$\mu_0 \varepsilon_0 = \frac{1}{c^2},$$

this becomes

$$\boxed{W_{C0}^{(\mathrm{in})} = \varepsilon_0 E_0^2 \frac{\omega^2 \pi L a^4}{16 c^2}.}$$

This expression represents only the magnetic energy inside the idealized cylindrical core. It is therefore a simplified estimate of the energy associated with the proposed intermediate configuration.

6.3 Geometrical condition

Within this approximation, complete energy conversion requires

$$W_{T0} = W_{C0}^{(\mathrm{in})}.$$

Substituting the expressions above,

$$\varepsilon_0 E_0^2 V_T = \varepsilon_0 E_0^2 \frac{\omega^2 \pi L a^4}{16 c^2}.$$

The common amplitude factor cancels, giving

$$\boxed{V_T = \frac{\omega^2 \pi L a^4}{16 c^2}.}$$

Using

$$\frac{\omega}{c} = \frac{2\pi}{\lambda},$$

this may also be written as

$$\boxed{V_T = \frac{\pi^3}{4} \frac{L a^4}{\lambda^2}.}$$

This equation should not yet be interpreted as a precise geometrical prediction of the model. Its numerical coefficient depends on the assumed field profiles and on the simplified cylindrical approximation. Its more important feature is the form of the result:

energy equivalence →
relation between geometry and frequency.

The field amplitude does not appear in the final relation.

6.4 Field energy outside the cylindrical core

The magnetic field associated with the effective displacement current is not confined to r<a. For r>a, the ideal cylindrical approximation gives

$$B_I(r) = \frac{\mu_0 I_{D0}}{2\pi r},$$

where

$$I_{D0} = J_{D0} \pi a^2.$$

If this field extends effectively to a radius b, its additional magnetic energy is

$$W_{C0}^{(\text{out})} = \int_a^b \frac{B_I^2}{2\mu_0} 2\pi r L \, dr,$$

which yields

$$W_{C0}^{(\text{out})} = \frac{\mu_0 I_{D0}^2 L}{4\pi} \ln\left(\frac{b}{a}\right).$$

Since

$$I_{D0} = J_{D0}\pi a^2,$$

the complete magnetic-energy estimate becomes

$$W_{C0} = W_{C0}^{(\text{in})} + W_{C0}^{(\text{out})},$$

or

$$\boxed{W_{C0} = \frac{\mu_0 I_{D0}^2 L}{16\pi}\left[1 + 4\ln\left(\frac{b}{a}\right)\right].}$$

The parameter b cannot be regarded as an arbitrary physical boundary. In a self-contained electromagnetic structure, the outer field distribution must emerge from the complete three-dimensional solution. The logarithmic term therefore demonstrates why the simplified numerical coefficient derived above cannot yet be regarded as fundamental.

6.5 What survives the geometrical approximation

Although the detailed geometrical coefficient remains uncertain, a more general property can already be identified.

The toroidal energy has the scaling

$$W_{T0} \propto E_0^2,$$

while, because

$$J_{D0} \propto E_0,$$

the magnetic energy associated with the displacement-current configuration also scales as

$$W_{C0} \propto J_{D0}^2 \propto E_0^2.$$

Thus, in any linear version of the model in which the geometrical dimensions do not themselves depend on amplitude, the energy-equivalence condition has the general form

$$E_0^2 F_T(\omega, \text{geometry}) = E_0^2 F_C(\omega, \text{geometry}).$$

For $E_0 \neq 0$, this reduces to

$$\boxed{F_T(\omega, \text{geometry}) = F_C(\omega, \text{geometry}).}$$

The cancellation of amplitude is therefore more general than the particular cylindrical approximation used above.

This is the central result of the present section. The numerical geometrical relation remains model-dependent, but the amplitude cancellation follows from the common quadratic scaling of electromagnetic energy in a linear field theory.

Consequently, the condition required for the proposed periodic transformation does not by itself select a unique value of E_0, B_0, or the total energy W. It constrains the relationship between the characteristic geometry and frequency.

This raises an important question. If the field amplitude is not fixed by the propagation condition, must an electromagnetic structure occupy the preferred energy W0 introduced in Section 3 in order to propagate?

The following section examines this distinction between propagation and optimal structural energy.

7. Propagation Independent of the Optimal Amplitude

The preceding section showed that, within the linear approximation of the proposed model, both the toroidal and displacement-current configurations have energies proportional to the square of the field amplitude. Their energy-equivalence condition can therefore be written in the general form

$$E_0^2 F_T(\omega, \text{geometry}) = E_0^2 F_C(\omega, \text{geometry}).$$

For any nonzero field amplitude, the common factor cancels,

$$F_T(\omega, \text{geometry}) = F_C(\omega, \text{geometry}).$$

Thus, the proposed condition for periodic transformation between the two field configurations constrains their geometry and frequency, but does not select a particular value of E_0 or B_0.

This result is consistent with a fundamental property of the vacuum Maxwell equations. They are linear in the electromagnetic fields. If

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t)$$

form a solution, then

$$\alpha\mathbf{E}(\mathbf{r}, t), \qquad \alpha\mathbf{B}(\mathbf{r}, t)$$

also form a solution for any constant scaling factor α. The frequency, wavelength, and propagation velocity remain unchanged, while the electromagnetic energy scales as

$$W \to \alpha^2 W.$$

The proposed torus–cylinder propagation mechanism therefore inherits this amplitude freedom.

7.1. Propagation and preferred structure

This result must be distinguished from the preferred structural state introduced in Section 3. There, W_0 was defined as the hypothetical energy corresponding to a stable minimum of the toroidal configuration.

If such a preferred state exists, it may be characterized by

$$W = W_0, \qquad B = B_{\text{opt}}.$$

However, the propagation condition derived above does not require

$$B = B_{\text{opt}}.$$

An electromagnetic structure may therefore, in principle, propagate with

$$B \neq B_{\text{opt}}$$

and

$$W \neq W_0.$$

The two concepts are consequently distinct:

propagation condition ≠ optimal structural condition

Propagation concerns whether the electromagnetic configuration can continuously reproduce its dynamical cycle. Structural optimization concerns which field configuration is energetically preferred.

7.2. A non-optimal propagating structure

Suppose an optimal structure has energy W_0 and characteristic amplitude B_{opt}. If another otherwise similar structure has energy

$$W = \alpha W_0,$$

then, provided that its effective geometry remains approximately unchanged,

$$B = \sqrt{\alpha}\, B_{\text{opt}}.$$

For example, if

$$W = 0.9 W_0,$$

the corresponding field amplitude is approximately

$$B \simeq \sqrt{0.9}\, B_{\text{opt}} \simeq 0.95 B_{\text{opt}}.$$

Such a structure would be non-optimal with respect to the hypothetical energy minimum, but the amplitude cancellation derived in Section 6 provides no reason for its propagation cycle to stop.

Its energy transformation would simply occur at the lower total energy:

$$W_T(t) = W\cos^2\omega t,$$

$$W_C(t) = W\sin^2\omega t,$$

with

$$W_T(t) + W_C(t) = W.$$

Thus, the periodic transformation can in principle operate at different overall energy scales.

7.3. What determines the optimal amplitude?

If the propagation equations do not determine B_{opt}, a preferred amplitude must originate from another property of the structure.

In the present model, the natural candidate is the hypothetical stability condition of the toroidal geometry,

$$\left.\frac{dW}{dR}\right|_{R_0} = 0, \qquad \left.\frac{d^2W}{dR^2}\right|_{R_0} > 0.$$

A particular combination of geometry and field amplitude could then correspond to the most stable configuration.

The important point is that such a minimum would represent a **structural preference**, not a minimum energy required for electromagnetic propagation.

This leads to the following hierarchy:

Maxwellian dynamics → possible propagation over a range of amplitudes

structural stability → possible preferred amplitude B_{opt}

The existence and value of Bopt remain to be derived from a complete three-dimensional field solution.

7.4. Isolated and interacting structures

The distinction becomes particularly important for an isolated electromagnetic structure. If such a structure is created with

$$W \neq W_0,$$

and no interaction allows it to gain or lose energy, there is no obvious mechanism forcing it immediately toward W_0. It may therefore remain in a non-optimal state while continuing its propagation cycle.

A different situation arises when neighboring electromagnetic structures interact. Their individual energies need no longer remain separately conserved, even though the total energy of the combined system does.

This opens the possibility that electromagnetic coupling could redistribute energy between neighboring structures and thereby allow some of them to approach the preferred state W_0.

The propagation analysis itself does not establish that such redistribution occurs, nor does it determine its direction. These questions require a separate model of electromagnetic coupling and stability.

The following section therefore examines whether a coherent sequence of neighboring toroidal structures could provide such an interaction mechanism.

8. Coherent Chains of Toroidal Structures

The preceding section showed that the proposed propagation condition does not require an electromagnetic structure to possess its preferred energy W_0. An isolated structure may therefore, in principle, propagate in a non-optimal state. A different situation arises when several electromagnetic structures overlap and interact.

Consider a sequence of toroidal structures aligned along a common propagation direction,

$$T_1,\ T_2,\ T_3, \ldots, T_N.$$

The fields of an electromagnetic structure do not terminate abruptly at a geometrical boundary. Even if most of its energy is concentrated within an effective toroidal region, weaker peripheral fields extend beyond it. Neighboring structures can therefore overlap.

This overlap provides a possible mechanism for electromagnetic coupling. The total fields in the overlap region are given by superposition,

$$\mathbf{E}_{\text{tot}} = \mathbf{E}_1 + \mathbf{E}_2,$$
$$\mathbf{B}_{\text{tot}} = \mathbf{B}_1 + \mathbf{B}_2.$$

The local electromagnetic energy density consequently contains not only the separate contributions of the two structures but also cross terms arising from their overlap. For the electric contribution,

$$|\mathbf{E}_1 + \mathbf{E}_2|^2 = E_1^2 + E_2^2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2,$$

with an analogous expression for the magnetic field.

The interaction therefore depends on the relative orientation and phase of the neighboring fields. This provides a classical electromagnetic basis for weak coupling between adjacent structures.

8.1. Coherence and coupling

In the present model, a coherent sequence is represented schematically as

$$T_1 \leftrightarrow T_2 \leftrightarrow T_3 \leftrightarrow \cdots \leftrightarrow T_N.$$

Coherence means that the relative phases of neighboring fields remain sufficiently well defined for their overlap to produce a systematic interaction. The toroidal structures should not be understood as completely independent objects merely placed next to one another. Their peripheral fields form part of one combined electromagnetic field.

Figure 4 illustrates this proposed coupling.

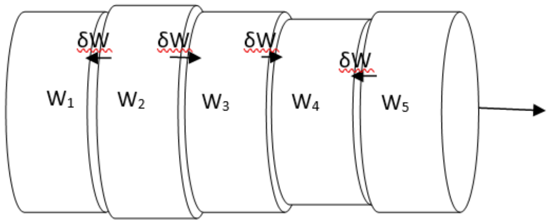

Figure 4. *Schematic representation of some neighboring toroidal field structures in a coherent sequence. The strongest fields are concentrated within each toroidal region, while weaker peripheral fields overlap. This overlap provides a possible electromagnetic coupling between neighboring structures and may allow energy exchange.*

The coupling may be weak because the overlapping fields are smaller than those in the central regions of the structures. Nevertheless, weak coupling can become significant when the phase relation persists over many oscillation cycles.

8.2. Energy exchange between neighboring structures

If two neighboring structures exchange electromagnetic energy, their individual energies need not remain constant. We may write schematically

$$\frac{dW_1}{dt} = -P_{12},$$
$$\frac{dW_2}{dt} = P_{12},$$

where P_{12} denotes the instantaneous power transferred between them.

Consequently,

$$\frac{d}{dt}(W_1 + W_2) = 0,$$

provided that no energy enters or leaves the two-structure system.

The exact expression for P_{12} has not yet been derived. It must ultimately follow from the overlapping fields and their Poynting flux. In general, it may depend on field amplitudes, geometry, separation, and relative phase,

$$P_{12} = P(\mathbf{E}_1, \mathbf{B}_1, \mathbf{E}_2, \mathbf{B}_2, \Delta\phi).$$

Thus, electromagnetic coupling provides a mechanism by which energy exchange is possible, but it does not by itself determine the final distribution of energy.

8.3. The role of the preferred energy W_0

Section 3 introduced the hypothesis that a toroidal structure may possess a preferred stable energy W_0. If such a stability minimum exists, coupling between neighboring structures introduces a new possibility.

Consider two structures with energies

$$W_1 = W_0 + \Delta W,$$

and

$$W_2 = W_0 - \Delta W.$$

Their total energy is

$$W_1 + W_2 = 2W_0.$$

If the coupled system tends toward configurations of greater stability, energy exchange could in principle lead to

$$W_1 \to W_0, \qquad W_2 \to W_0.$$

No energy needs to be created or destroyed; only its distribution between the structures changes.

This example illustrates the possible role of coupling, but it does not establish that W_0 will necessarily be selected. In a purely linear electromagnetic system, a continuum of amplitudes remains possible. A preferred final energy requires the additional stability mechanism proposed in Section 3.

Thus,

coupling permits energy redistribution

whereas

structural stability would have to select W_0

These are two distinct requirements.

8.4. An energy deficit in a coherent chain

The distinction becomes especially important when the total energy of a chain is not an integer multiple of W_0.

Suppose

$$W_{\text{chain}} = (N - \alpha)W_0, \qquad 0 < \alpha < 1.$$

Several final distributions are then mathematically possible. In a linear system, the deficit may simply remain continuously distributed among the N structures.

If, however, W_0 corresponds to a pronounced nonlinear stability minimum, another possibility arises: energy redistribution may bring some structures closer to W_0 while concentrating the remaining deficit in fewer structures.

Schematically,

$$W_1, W_2, \dots, W_N \quad \longrightarrow \quad W_0, W_0, \dots, W_{\text{res}}.$$

Here

$$W_{\text{res}} < W_0$$

represents a remaining non-optimal configuration.

If a lower stability threshold existed, a sufficiently under-energized structure might cease to remain localized as an identifiable toroidal configuration. Its remaining energy would still have to be conserved and transferred into other electromagnetic degrees of freedom.

This possibility provides a physical interpretation of the earlier intuitive idea that a coherent chain might effectively “sacrifice” one structure so that others remain near their preferred state. However, such behavior **does not follow from linear Maxwell equations alone**. It requires a nonlinear stability law that has not yet been derived.

8.5. Coherence as an environment for energy organization

The proposed role of coherence can therefore be stated more modestly and more precisely.

An isolated structure has no neighboring electromagnetic structure with which it can directly redistribute energy. If it propagates at

$$W \neq W_0,$$

it may remain non-optimal.

A coherent chain provides an interaction environment in which individual energies can change while total energy remains conserved:

$$\sum_{n=1}^{N} W_n = W_{\text{chain}} = \text{const.}$$

If a preferred structural energy W_0 exists independently, coupling could then allow the system to explore configurations closer to that preferred state.

The proposed sequence is therefore

field overlap → electromagnetic coupling → energy redistribution

followed, only if nonlinear structural stability exists, by

energy redistribution → preference for W_0

This distinction is essential. Coherence and field overlap provide a plausible mechanism for interaction; they do not themselves establish energy quantization.

The possibility that a preferred structural energy combined with coherent energy redistribution could produce discrete-looking energy-transfer events motivates the following section. There, the model is compared with the experimentally established relation between frequency and energy transfer in the photoelectric effect.

9. Possible Relation to Energy Quantization and the Photoelectric Effect

The preceding sections lead to a distinction between continuous electromagnetic propagation and a possible preferred structural energy. Within the proposed model, propagation itself does not select a unique field amplitude or energy. If a preferred energy W_0 exists, it must therefore arise from the stability of the electromagnetic structure rather than from the propagation condition.

Section 8 introduced a second ingredient: coherent coupling may allow neighboring structures to exchange energy. Taken together, these two hypotheses suggest a possible route by which continuously propagating electromagnetic fields could nevertheless exhibit preferred energies during interaction with matter.

Schematically,

continuous propagation →
preferred structural state →
energy redistribution →
discrete energy transfer.

This possibility can be compared with the experimentally established behavior of the photoelectric effect.

9.1 Experimental constraint

The photoelectric effect places a strong requirement on any model of electromagnetic energy transfer. The maximum kinetic energy of an emitted electron is determined by the frequency of the incident radiation,

$$K_{\max} = h\nu - \Phi,$$

where h is Planck's constant and Φ is the work function of the material.

At a fixed frequency, increasing the radiation intensity primarily increases the number of emitted electrons rather than their maximum kinetic energy. Furthermore, photoemission does not occur below the threshold frequency

$$\nu_0 = \frac{\Phi}{h},$$

regardless of the classical intensity of the incident radiation.

Any field-based model intended to account for this phenomenon must reproduce these observations quantitatively.

9.2 A possible preferred energy

In the present model, the natural candidate for a characteristic transferable energy is the preferred structural energy W_0 introduced in Section 3.

If W_0 depends on frequency,

$$W_0 = W_0(\nu),$$

then interaction with matter could, in principle, occur preferentially in units associated with this stable field configuration.

For the model to reproduce the experimentally established photoelectric relation, however, it must ultimately yield

$$\boxed{W_0(\nu) = h\nu.}$$

This relation is **not derived in the present model**. It represents one of its principal quantitative requirements.

The distinction is important. The model does not explain energy quantization merely by introducing a preferred energy W_0. It must explain why that energy is proportional to frequency over the observed electromagnetic spectrum and why the proportionality constant has the universal value h.

9.3 Frequency and field dynamics

The proposed torus–cylinder transformation nevertheless identifies a field quantity that depends directly on frequency.

For

$$B(t) = B_0 \cos \omega t,$$

the maximum rate of magnetic-field variation is

$$\left|\frac{dB}{dt}\right|_{\max} = \omega B_0.$$

Similarly,

$$\left|\frac{dE}{dt}\right|_{\max} = \omega E_0,$$

and therefore

$$J_{D0} = \varepsilon_0 \omega E_0.$$

Thus, frequency directly controls the rate at which the electromagnetic configuration changes.

This raises the possibility that interaction with a bound electron may depend not only on field magnitude but also on the rate of field variation. A threshold condition might therefore involve a quantity such as

$$\left|\frac{dB}{dt}\right|_{\max},$$

or an equivalent dynamical property of the complete electromagnetic field.

Such a condition could schematically be written as

$$\left|\frac{dB}{dt}\right|_{\max} \geq \left|\frac{dB}{dt}\right|_{\mathrm{crit}}$$

At present, this should be regarded only as a possible interaction mechanism. It is not equivalent to the experimentally observed frequency threshold, because

$$\left|\frac{dB}{dt}\right|_{\max} = \omega B_0$$

depends on both frequency and amplitude. The photoelectric threshold, by contrast, cannot generally be replaced by a simple amplitude-dependent condition.

A successful field model would therefore have to explain how the structural dynamics remove or constrain this amplitude dependence.

9.4 Intensity and number of transferable structures

If the preferred structural energy were eventually shown to satisfy

$$W_0 = h\nu,$$

then the different roles of frequency and intensity could receive a geometrical interpretation.

For a coherent sequence containing N structures near the preferred state,

$$W_{\text{chain}} \simeq NW_0.$$

Increasing intensity at fixed frequency could then correspond primarily to increasing the number or density of available structures rather than increasing the preferred energy of each structure.

Schematically,

$$\text{frequency} \rightarrow W_0(\nu),$$

whereas

intensity → number of available energy-transfer events.

This would resemble the experimentally observed distinction between the energy of individual photoelectric events and their rate.

However, this interpretation depends critically on the still-unproven existence of stable individual structures with preferred energy W_0 .

9.5 Interaction with a bound electron

Suppose a bound electron requires energy Φ to escape from a material. If an electromagnetic structure transfers energy W_0 in a localized interaction, energy conservation gives

$$W_0 = \Phi + K.$$

Therefore,

$$K = W_0 - \Phi$$

If the structural theory independently produced

$$W_0 = h\nu,$$

the conventional photoelectric equation would follow,

$$K = h\nu - \Phi.$$

The experimentally verified equation would therefore remain unchanged. What would differ is its physical interpretation: hν would represent the preferred transferable energy of a stable electromagnetic field configuration.

This interpretation does not require the present analysis to assume a point-like electromagnetic particle. It asks instead whether localized discrete energy transfer could emerge from the organization and stability of the electromagnetic field itself.

9.6 What the model must still explain

The photoelectric effect provides a particularly demanding test because reproducing only the relation

$$W_0 \propto \nu$$

would not be sufficient.

A complete theory would need to account simultaneously for the universal coefficient h, the threshold frequency, the maximum photoelectron kinetic energy, the dependence of emission rate on intensity, and the localized character and timing of individual energy-transfer events.

The present model does not yet provide these derivations.

Its more limited contribution is to suggest a possible separation between three stages:

$$\boxed{\text{continuous electromagnetic propagation}}$$

$$\boxed{\text{selection of a preferred structural energy}}$$

$$\boxed{\text{localized energy transfer to matter.}}$$

Whether the first two can produce the experimentally observed third remains an open question.

The decisive theoretical step is therefore the independent derivation of

$$\boxed{W_0(\nu) = h\nu}$$

from the geometry and dynamics of the proposed electromagnetic structure. Until such a derivation is obtained, the connection between the toroidal model and quantum energy transfer remains a hypothesis rather than a result.

10. Discussion and Open Problems

The model developed in this paper combines established relations of classical electrodynamics with additional hypotheses concerning the three-dimensional organization of electromagnetic fields. Throughout the preceding sections, an effort has been made to distinguish these two levels.

The Maxwell equations provide the underlying dynamical framework. In particular,

$$\nabla \cdot \mathbf{B} = 0$$

permits closed magnetic-field configurations, while

$$\mathbf{J}_D = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

establishes the quarter-period phase relation between a sinusoidally varying electric field and displacement current. The quadratic dependence of electromagnetic energy on field amplitude also explains why the amplitude cancels from the energy-equivalence condition developed in Section 6.

These results do not, however, establish the proposed global field geometry. The toroidal organization, its transformation into an approximately cylindrical displacement-current configuration, and the existence of a preferred stable energy W_0 remain hypotheses that require further theoretical development.

10.1 Three-dimensional field solution

The most fundamental open problem is the construction of explicit fields

$$\mathbf{E}(\mathbf{r}, t), \qquad \mathbf{B}(\mathbf{r}, t),$$

that exhibit the proposed geometry while satisfying the complete Maxwell equations and appropriate boundary conditions.

Such a solution would determine whether the toroidal and cylindrical configurations can represent successive phases of one continuous electromagnetic field rather than merely a useful geometrical picture.

It would also allow the proposed energy transformation to be tested directly using the conventional electromagnetic energy density and Poynting's theorem.

10.2 Stability and the preferred energy

The second problem concerns stability. The model assumes that electromagnetic stresses may produce a preferred toroidal configuration characterized by

$$R_0, \qquad W_0, \qquad B_{\text{opt}}.$$

The existence of such a stable configuration has not yet been demonstrated.

A complete calculation using the Maxwell stress tensor must determine whether the electromagnetic field can possess a stable energy minimum of the form

$$\left.\frac{dW}{dR}\right|_{R_0} = 0, \qquad \left.\frac{d^2W}{dR^2}\right|_{R_0} > 0.$$

This question is central because the later discussion of preferred energies and coherent redistribution depends on the existence of W0

.

10.3 Coherence and energy redistribution

Field overlap provides a classical mechanism for interaction between neighboring electromagnetic structures. However, linear electromagnetic coupling alone does not select a particular energy.

The proposed tendency of coherent structures toward W therefore requires an additional nonlinear stability mechanism. A quantitative theory must derive the coupling law and show under what conditions energy redistribution favors the proposed stable configuration.

10.4 Relation to energy quantization

The strongest quantitative requirement arises from the experimentally established relation

$$E = h\nu.$$

If the preferred structural energy proposed in this paper is to account for discrete electromagnetic energy transfer, the model must independently derive

$$\boxed{W_0(\nu) = h\nu.}$$

Neither the linear dependence on frequency nor the value of Planck's constant may be assumed. They must emerge from the geometry and dynamics of the field structure.

The photoelectric effect should therefore be regarded at this stage as a **constraint on the model rather than a result of the model**.

10.5 Experimental test

Finally, a geometrical interpretation becomes physically distinguishable from the conventional description only if it leads to a measurable prediction.

The decisive future test would have the form

$$\text{toroidal model} \to A,$$

while the conventional description predicts

$$\text{standard theory} \to B, \qquad A \neq B.$$

Such a prediction might involve the internal field geometry, interaction between coherent structures, or the behavior of non-optimal localized electromagnetic configurations. No decisive experiment of this kind has yet been identified in the present work.

The present analysis therefore establishes a framework rather than a completed theory. Its central conceptual distinction is

Propagation ≠ structural stability ≠ energy quantization

The first is compatible with continuous-amplitude Maxwellian dynamics. The second requires demonstration of a stable three-dimensional electromagnetic structure. The third would require showing that such stability, together with electromagnetic coupling, leads quantitatively to the observed relation E=hv.

11. Conclusions

This paper has explored a possible three-dimensional interpretation of electromagnetic wave propagation based on a toroidal organization of the electric and magnetic fields. The model does not modify Maxwell's equations; rather, it asks whether their local field dynamics can be associated with a particular global electromagnetic geometry.

The proposed description distinguishes two complementary configurations,

$$(E, B_T)_{\text{torus}} \rightleftarrows (I_D, B_I)_{\text{cylinder}},$$

where the second configuration is associated with Maxwell's displacement current. For sinusoidal variation, the electric field and displacement current are shifted by one quarter of a period. This provides the basis for interpreting propagation as a continuous geometrical transformation rather than as the translation of an unchanged toroidal object through space.

Energy conservation requires the maximum energies associated with the two proposed configurations to be equal. Within the simplified geometrical model, both energies scale with the square of the field amplitude,

$$W_T \propto E_0^2, \qquad W_C \propto E_0^2.$$

Consequently, the common amplitude factor cancels from the energy-equivalence condition. The resulting constraint concerns geometry and frequency rather than a particular field amplitude.

This leads to one of the central conclusions of the analysis: **the proposed propagation condition does not require the electromagnetic structure to possess an optimal amplitude or energy.** A preferred energy W_0, if such a state exists, must therefore originate from structural stability rather than from propagation itself.

The model further considers whether overlapping fields in a coherent sequence could allow neighboring structures to exchange energy. Such electromagnetic coupling can provide a mechanism for energy redistribution, but selection of a preferred energy W_0 would require an additional nonlinear stability mechanism that has not yet been derived.

This leads to the conceptual distinction

Propagation ≠ structural stability ≠ energy quantization

Within the proposed interpretation, continuous-amplitude propagation is associated with Maxwellian field dynamics, while a preferred energy would have to emerge from the stability of the three-dimensional field structure. Discrete energy transfer could then become possible only if such preferred states interact with matter in a correspondingly localized manner.

The connection with quantum phenomena therefore remains an open question. In particular, explaining the photoelectric effect would require the model to derive independently

$$\boxed{W_0(\nu) = h\nu,}$$

rather than introducing this relation as an assumption.

The principal theoretical task is consequently the construction of an explicit three-dimensional electromagnetic field solution exhibiting the proposed toroidal dynamics. Such a solution must satisfy Maxwell's equations, conserve energy according to Poynting's theorem, and demonstrate the proposed structural stability through the electromagnetic stress distribution.

Until these requirements are met, the toroidal model should be regarded as a geometrical hypothesis rather than a completed theory of electromagnetic radiation. Its value lies in posing a specific physical question: **can propagation, structural stability, and discrete energy transfer arise as distinct properties of a self-organized electromagnetic field?**

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